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International Journal of Heat and Mass Transfer 44 (2001) 1455–1463

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Estimation of surface temperature in two-dimensional inverse heat conduction problems

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Received 15 October 1999; received in revised form 22 June 2000

Abstract

A hybrid numerical algorithm of the Laplace transform technique and finite-difference method with a sequential-in-time concept and the least-squares scheme is proposed to predict the unknown surface temperature in two-dimensional inverse heat conduction problems. In the present study, the expression of the surface temperature is unknown a priori. The whole time domain is divided into several analysis sub-time intervals and then the surface temperature in each analysis interval is estimated. To enhance the accuracy and efficiency of the present method, a good comparison between the present estimations and previous results is demonstrated. Results show that good estimations on the surface temperature can be obtained from the knowledge of the transient temperature recordings only at a few selected locations even for the case with measurement errors. It is worth mentioning that the unknown surface temperature can be accurately estimated even though the thermocouples are located far from the estimated surface. Due to the application of the Laplace transform technique, the unknown surface temperature distribution can be estimated from a specific time.

1. Introduction

Quantitative studies of the heat transfer processes occurring in the industrial applications require accurate knowledge of the thermal properties of the material and surface conditions. Practically, measurements are often made of temperature and displacement, etc. Thereafter, these measurements are fitted and then physical quantities and surface conditions may be estimated from these curve-fitted measurements. Such problems are called inverse problems and have become an interesting subject recently. To date, various methods have been developed for the analysis of the inverse heat conduction problems involving the estimation of surface conditions from measured temperatures inside the material [1–12]. However, most analytical and numerical methods were only used to deal with one-dimensional inverse heat conduction problems (IHCP). A few works were presented for two- or three-dimensional IHCP because the difficulty of these problems was more pronounced than one-dimensional IHCP.

Sparrow et al. [3] and Woo and Chow [4] applied the Laplace transform method to one-dimensional IHCP. Their results have good accuracy only for small value of time. Thus the application of their methods [3,4] was limited. Imber [5] obtained an analytical solution of the two-dimensional IHCP. Busby and Trujillo [6] applied the dynamic programing method to investigate the two-dimensional IHCP. The boundary element method in conjunction with the Beck's sensitivity analysis and least-squares method was presented for the solution of two-dimensional linear IHCP by Zabaras and Liu [7]. Yang and Chen [8] applied the finite-difference method in conjunction with the linear least-squares method to estimate the one-sided boundary condition in two-dimensional IHCP. In their work [8], the unknown surface temperature was parametrized and its functional form was also given a priori. Thus a few measurement locations can be sufficient to estimate the unknown surface temperature. However, the effect of the measurement errors on the estimated surface temperature

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Nomenclature

C_i	undetermined coefficient
$F(y,t)$	estimated function
$\{f\}$	force matrix
J	number of thermocouples
$[k]$	global conduction matrix
L	side length of a square plane plate
ℓ_x	distance between two nodes in the x-direction
ℓ_y	distance between two nodes in the y-direction
M	number of discrete measurement times
n	number of measurements
n_x	number of nodes in the x-direction
n_y	number of nodes in the y-direction
s	Laplace transform parameter
T	temperature
t	dimensionless time
t_f	dimensionless final time
x, y	dimensionless spatial coordinates

Greek symbols

α	thermal diffusivity
$\tilde{T}$	transformed dimensionless temperature
$\{\tilde{T}\}$	global dimensionless temperature matrix in the transform domain
σ^*	standard deviation of the mean
ω	averaged random error

cannot be neglected. Yang [9] and Hsu et al. [10] also applied the same inverse technique to analyze other two-dimensional IHCP. In order to estimate the heat flux at the surface from experimentally measured transient temperature data, Osman et al. [11] presented a combined function specification and regularization method with a sequential-in-time concept.

Chen and Chang [12] have ever used the hybrid method of the Laplace transform technique and finite-difference method to estimate the unknown surface temperature in one-dimensional IHCP using measured nodal temperatures inside the material at any specific time without measurement errors. In general, there exists an optimum combination of the time step and the mesh size for error control of the numerical solution. Thus the present study applies the Laplace transform technique and finite-difference method with a sequential-in-time concept to estimate the unknown surface temperature in two-dimensional IHCP. The functional form of the surface temperature is unknown a priori in the present study. In order to evidence the accuracy of the estimates, more temperature measurements are needed than the number of unknowns. In performing the numerical simulation of the present study, the whole time domain is divided into several analysis sub-time intervals and then the surface temperature in each analysis interval is estimated. The computational procedure for the estimation of the surface temperature is performed repeatedly until the sum of the squares of the deviations between the calculated and measured temperatures becomes minimum. In order to show the efficiency of the present method in estimating the surface temperature from temperature measurements, a comparison between the present estimates and results given by Yang and Chen [8] is also made.

In experiments, the measurement of temperature is, in general, somewhat inaccurate. This may be due to human error, but more often, it is due to inherent limitations in the equipment being used to make the measurements. In the inverse heat conduction problem, slight inaccuracies in the measured interior temperatures can affect the accuracy of estimated surface condition. Thus the effect of measurement errors on the estimation of the surface temperature will be investigated in the present analysis.

2. Mathematical formulation

Consider a square plane plate with the length of the side L. The initial temperature is T_0^*. For time $t^* > 0$, the boundaries at $x^* = 0$ and $y^* = L$ keep insulated. The surface temperature at $y^* = 0$ is isothermal and is equal to T_0^*. For the direct heat conduction problem, the temperature field in the plane plate as a function of space and time can be determined by providing the surface temperature at $x^* = L$. Conversely, the surface temperature at $x^* = L$ needs to be estimated unless additional information on temperature in the slab is given. For convenience of numerical analysis, the following dimensionless parameters are introduced

$$T = \frac{T^* - T_0^*}{T_0^*}, \quad x = \frac{x^*}{L}, \quad y = \frac{y^*}{L}, \quad t = \frac{\alpha t^*}{L^2}, \tag{1}$$

where α is the thermal diffusivity.

In order to compare with the results of Yang and Chen [8], the dimensionless form of a two-dimensional heat conduction problem in the Cartesian coordinate system with the dimensionless parameters in Eq. (1), as shown in Fig. 1, is illustrated

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \quad \text{in } 0 < x < 1, \ 0 < y < 1, \ 0 < t \leqslant t_f \tag{2}$$

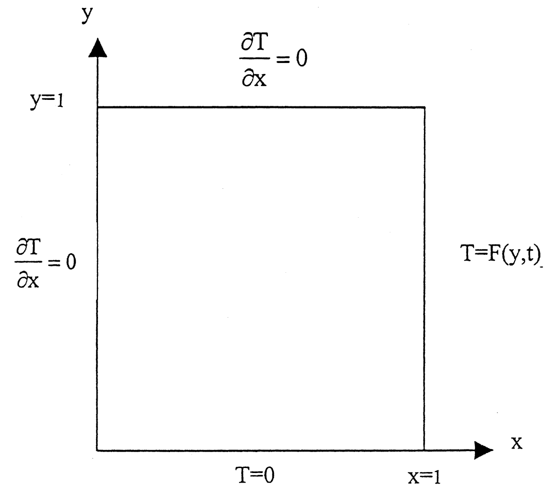

Fig. 1. Geometry of two-dimensional plane plate.

with the dimensionless boundary conditions

$$T = F(y,t) \quad \text{at } x = 1, \tag{3}$$

$$T = 0 \quad \text{at } y = 0, \tag{4}$$

$$\frac{\partial T}{\partial x} = 0 \quad \text{at } x = 0, \tag{5}$$

$$\frac{\partial T}{\partial y} = 0 \quad \text{at } y = 1 \tag{6}$$

and the dimensionless initial condition

$$T = 0 \quad \text{for } t = 0, \tag{7}$$

where t_f is the dimensionless final time for temperature measurements. The continuous surface temperature function $F(y,t)$ in Eq. (3) should be the temporal and spatial distribution. It is unknown and will be estimated from some temperature measurements.

To estimate the unknown function $F(y,t)$, the additional information of discrete temperature measurements is required. Thus the temperature histories at some locations are measured in the plane plate. It is assumed that J thermocouples are used to record the temperature information at these selected locations, as shown in Table 1. The temperature histories taken from the thermocouples at successive specific dimensionless time t_m are denoted by $T_{i,m}^{\text{mea}}, i = 1,\ldots,J$ and $m = 1,\ldots,M$, where M denotes the number of the discrete measurement times. The temperature histories measured by these thermocouples will be used to estimate $F(y,t)$.

The simulated data of measurements, $T_{i,m}^{\text{mea}}$, can be obtained by adding small random errors to the exact values from the solution to the direct problem. $T_{i,m}^{\text{mea}}$ used in the present inverse analysis can be expressed as

$$T_{i,m}^{\text{mea}} = T_{i,m}^{\text{exa}}(1+\omega), \quad m = 1,\ldots,M \tag{8}$$

Table 1
Measurement locations of the present study and Yang and Chen [8]

Yang and Chen [8]	Present study	
	Case A	Case B
(0.5, 0.4)	(0.6, 0.1)	(0.1, 0.2)
(0.5, 0.6)	(0.5, 0.3)	(0.1, 0.4)
(0.4, 0.5)	(0.4, 0.7)	(0.1, 0.6)
(0.5, 0.4)	(0.6, 0.1)	(0.1, 0.2)
(0.5, 0.6)	(0.5, 0.3)	(0.1, 0.4)
(0.4, 0.5)	(0.4, 0.7)	(0.1, 0.6)
(0.6, 0.5)	(0.5, 0.9)	(0.1, 0.8)

where ω represents the averaged random error and is assumed to be within −0.05 to 0.05 in the present study. σ^* is the standard deviation of the mean with respect to the exact data and is defined as [13]

$$\sigma^* = \left[\sum_{m=1}^{M}(T_{i,m}^{\text{mea}} - T_{i,m}^{\text{exa}})^2\right]^{1/2} \Big/ M, \quad m = 1,\ldots,M. \tag{9}$$

In practical applications, the actual measured profiles often exhibit random oscillations owing to measurement errors. Thus a polynomial function can be used to fit these measured data by using the least-squares scheme [13].

In order to remove the time-dependent terms from the governing differential equation (2) and boundary conditions (3)–(6) with the initial condition (7), the method of the Laplace transform is employed [12,14,15].

The Laplace transform of a function $\phi(t)$ is defined as follows:

$$\tilde{\phi}(s) = \int_0^{\infty} \phi(t)\mathrm{e}^{-st}\,\mathrm{d}t, \tag{10}$$

where s is the Laplace transform parameter.

The Laplace transform of equations (2)–(6) gives

$$\frac{\partial^2 \tilde{T}}{\partial x^2} + \frac{\partial^2 \tilde{T}}{\partial y} - s\tilde{T} = 0 \quad \text{in } 0 < x < 1, \; 0 < y < 1 \tag{11}$$

and

$$\tilde{T} = \tilde{F}(y,s) \quad \text{at } x = 1, \tag{12}$$

$$\tilde{T} = 0 \quad \text{at } y = 0, \tag{13}$$

$$\frac{\partial \tilde{T}}{\partial x} = 0 \quad \text{at } x = 0, \tag{14}$$

$$\frac{\partial \tilde{T}}{\partial y} = 0 \quad \text{at } y = 1. \tag{15}$$

The discretized forms of Eqs. (11)–(15) that can be obtained by using the central difference approximation are, respectively, given as

$$\frac{\tilde{T}_{j+1,k} - 2\tilde{T}_{j,k} + \tilde{T}_{j-1,k}}{\ell_x^2} + \frac{\tilde{T}_{j,k+1} - 2\tilde{T}_{j,k} + \tilde{T}_{j,k-1}}{\ell_y^2} = s\tilde{T}_{j,k},$$
$$j = 1,2,\ldots,n_x, \quad k = 1,2,\ldots,n_y \tag{16}$$

and

$$\tilde{T}_{n_x,k} = \tilde{F}((k-1)\ell_y, s), \quad k = 1,2,\ldots,n_y, \tag{17}$$

$$\tilde{T}_{j,1} = 0, \quad j = 1,2,\ldots,n_x, \tag{18}$$

$$\tilde{T}_{0,k} = \tilde{T}_{2,k}, \quad k = 1,2,\ldots,n_y, \tag{19}$$

$$\tilde{T}_{j,n_y-1} = \tilde{T}_{j,n_y+1}, \quad j = 1,2,\ldots n_x, \tag{20}$$

where n_x and n_y indicate the number of nodes along x- and y-directions, respectively. ℓ_x and ℓ_y, respectively, designate the distance between two neighboring nodes in the x- and y-directions and are defined by $\ell_x = 1/(n_x - 1)$ and $\ell_y = 1/(n_y - 1)$.

The rearrangement of equations (16)–(20) gives the following vector matrix equation:

$$[k]\{\tilde{T}\} = \{f\}, \tag{21}$$

where $[k]$ is an $n_x \times n_y$ matrix, $\{\tilde{T}\}$ is an $n_x \times 1$ matrix representing the unknown dimensionless nodal temperatures in the s domain and $\{f\}$ is an $n_x \times 1$ matrix. The Gaussian elimination algorithm and the numerical inversion of Laplace transform [16] are applied to invert the temperature $\tilde{T}$ in the s domain to that in the physical domain. The advantage of the present method is that the estimation of the unknown surface temperature at a specific time does not need to proceed with step-by-step computation from the initial time $t = 0$.

It is difficult to apply a polynomial function to fit a unknown function $F(y,t)$ for the whole time domain considered. Thus the time domain $t_0 \leqslant t \leqslant t_f$ will be divided into some analysis ranges where t_0 is the initial measurement time. Owing to the application of the Laplace transform in the present study, t_0 is not always the initial time. This implies that the estimation of the unknown surface temperature is carried out by discretizing the continuous function $F(y,t)$ in Eq. (3). Under this circumstance, a sequential-in-time procedure is introduced to estimate the unknown surface temperature. Assume that the dimensionless measurement time step Δt_e is $\Delta t_e = (t_f - t_0)/M$. The discrete time coordinate t_m is $t_m = t_0 + m\Delta t_e$ $(m = 1,2,\ldots,M)$. Each analysis interval in the present study is assumed to be $t_{m-1} \leqslant t \leqslant t_m$. The unknown surface temperature on each analysis interval can be approximated by a polynomial of degree $(p-1)$ in y and a polynomial of degree 2 in t before performing the inverse calculation. On the other hand, $F(y,t)$ can be expressed as

$$F(y,t) = \sum_{i=1}^{p} (C_{2i-1} + C_{2i}t) y^{i-1}, \tag{22}$$

where C_{2i-1} and C_{2i} are the unknown coefficients and are estimated simultaneously for each analysis interval. To evidence the accuracy and reliability of the estimates, the number of thermocouples, J, may be greater than the p value.

The least-squares minimization technique is applied to minimize the sum of the squares of the deviations between the calculated temperatures and curve-fitted temperature measurements taken from the ith thermocouple at $t = t_{m-1}$ and $t = t_m$. The error in the estimates $E(C_1, C_2, \ldots, C_{2p})$

$$E(C_1, C_2, \ldots, C_{2p}) = \sum_{n=m-1}^{m} \sum_{i=1}^{p} \left[T_{i,n}^{\text{cal}} - T_{i,n}^{\text{cur}} \right]^2$$
$$\text{for } m = 1,2,\ldots,M \tag{23}$$

is to be minimized. $T_{i,n}^{\text{cur}}$ is obtained from the curve- fitted profile of temperature measurements taken from the ith thermocouple at $t = t_n$. The estimated values of $C_j, j = 1,2,\ldots,2p$, are determined until the value of $E(C_1, C_2, \ldots, C_{2p})$ is minimum. The computational procedures for estimating the unknown coefficients $C_j, j = 1,2,\ldots,2p$, are described as follows.

First, the initial guesses of $C_j, j = 1,2,\ldots,2p$, are chosen. Accordingly, the calculated temperature taken from the ith thermocouple at $t = t_n, T_{i,n}^{\text{cal}}$, can be determined from Eq. (21). Deviations of $T_{i,n}^{\text{cur}}$ and $T_{i,n}^{\text{cal}}$ at $t = t_n$ are expressed as

$$e_{i,n} = T_{i,n}^{\text{cal}} - T_{i,n}^{\text{cur}}$$
$$\text{for } i = 1,2,\ldots,p \text{ and } n = m-1, m. \tag{24}$$

The new calculated temperature $T_{i,n}^{\text{cal},j}$ can be expanded in a first-order Taylor series as

$$T_{i,n}^{\text{cal},j} = T_{i,n}^{\text{cal}} + \sum_{j=1}^{2p} \frac{\partial T_{i,n}}{\partial C_j} \, \mathrm{d}C_j$$
$$\text{for } i = 1,2,\ldots p \text{ and } n = m-1, m. \tag{25}$$

The new estimated coefficient C_j^* is given in order to obtain the derivative $\partial T_{i,n}/\partial C_j$ and is expressed as

$$C_j^* = C_j + d_j \delta_{jk} \quad \text{for } j,k = 1,2,\ldots,2p, \tag{26}$$

where $d_j = C_j^* - C_j$ denotes the correction. The symbol δ_{jk} is Kronecker delta and is defined as

$$\delta_{jk} = \begin{cases} 1 & \text{if } j = k, \\ 0 & \text{if } j \neq k. \end{cases} \tag{27}$$

Accordingly, the new calculated temperature $T_{i,n}^{\text{cal},j}$ with respect to C_j^* given by Eq. (26) can be determined from Eq. (21). Deviations of $T_{i,n}^{\text{cal},j}$ and $T_{i,n}^{\text{cur}}$ can be defined as

$$e_{i,n}^j = T_{i,n}^{\text{cal},j} - T_{i,n}^{\text{cur}} \quad \text{for } i = 1,2,\ldots,p \text{ and } n = m-1, m. \tag{28}$$

The finite difference representation of the derivative $\partial T_{i,n}/\partial C_j$ can be expressed as

$$\omega_{i,n}^{j} = \frac{\partial T_{i,n}}{\partial C_j} = \frac{T_{i,n}^{\mathrm{cal},j} - T_{i,n}^{\mathrm{cal}}}{C_j^* - C_j} \quad \text{for } i = 1,2,\ldots,p \text{ and } n = m-1, m. \tag{29}$$

The function $\omega_{i,n}^{j}$ in conjunction with Eqs. (24), (26) and (28) can be written as

$$\omega_{i,n}^{j} = (e_{i,n}^{j} - e_{i,n})/d_j \quad \text{for } i = 1,2,\ldots,p \text{ and } n = m-1, m. \tag{30}$$

Thus Eq. (25) can be rewritten as

$$T_{i,n}^{\mathrm{cal},j} = T_{i,n}^{\mathrm{cal}} + \sum_{j=1}^{2p} \omega_{i,n}^{j} d_j^* \quad \text{for } i = 1,2,\ldots,p \text{ and } n = m-1, m, \tag{31}$$

where $d_j^* = dC_j$ denotes the new correction for the values of C_j.

Substituting Eq. (31) into Eq. (28) in conjunction with Eq. (24) yields

$$\boldsymbol{e}_{i,n}^{j} = \boldsymbol{e}_{i,n} + \sum_{j=1}^{2p} \omega_{i,n}^{j} d_j^* \quad \text{for } i = 1,2,\ldots,p \text{ and } n = m-1, m. \tag{32}$$

As shown in Eq. (23), the error in the estimates $E(C_1 + \Delta C_1, C_2 + \Delta C_2, \ldots, C_{2p} + \Delta C_{2p})$ can be expressed as

$$\boldsymbol{E} = \sum_{n=m-1}^{m} \sum_{i=1}^{p} (e_{i,n}^{j})^2. \tag{33}$$

To yield the minimum value of $\boldsymbol{E}$ with respect to C_j, differentiating $\boldsymbol{E}$ with respect to the new correction d_j^* will be performed. Thus the correction equations for the values of C_j can be expressed as

$$\sum_{j=1}^{2p} \sum_{n=m-1}^{m} \sum_{k=1}^{p} \omega_{k,n}^{i} \omega_{k,n}^{i} d_j^* = -\sum_{n=m-1}^{m} \sum_{j=1}^{p} \omega_{j,n}^{i} e_{j,n}, \quad i = 1,2,\ldots,2p. \tag{34}$$

Eq. (34) is a set of $2p$ algebraic equations for the new corrections. The new correction d_j^* can be obtained by solving Eq. (34). Furthermore, the new estimated coefficients can also be determined. The above procedures are repeated until the differences between $T_{i,n}^{\mathrm{cur}}$ and $T_{i,n}^{\mathrm{cal}}$ are all less than 10^{-4}.

3. Results and discussion

All the computations are performed on the PC. The present numerical results are obtained by using $t_0 = 0.01$, $\Delta t_e = 0.01$, $p = 3$ (or $p = 4$) and $\ell_x = \ell_y = 0.1$. The number of iterations is about four times for each analysis interval using the present method. In order to show the validity of the present method, the present results obtained by using three-point and four-point measurements are compared with those of Yang and Chen [8]. The unknown boundary condition shown in the work of Yang and Chen [8] is illustrated as follows:

$$\begin{aligned} T(1,y,t) &= F(y,t) \\ &= 0.1 + 0.2t + 0.3t^2 + 0.4t^3 + 0.1y \\ &\quad + 0.2y^2 + 0.3y^3. \end{aligned} \tag{35}$$

Yang and Chen [8] did not investigate the effect of the measurement locations on the estimates. Thus the present study uses different sets of the measurement locations to predict the surface temperature and investigates the effect of the measurement locations on the estimates. Table 1 shows the measurement locations given by the present study and Yang and Chen [8]. The measurement locations of Case A in Table 1 are in the neighborhood of the measurement locations given by Yang and Chen [8]. The difference of the present estimation using the measurement locations of Yang and Chen [8] and Case A in Table 1 is small. The present estimation using the measurement locations of Yang and Chen [8] have faster convergence than those using the measurement locations of Case A in each analysis interval. To avoid a misunderstanding, the present estimates using the measurement locations of Yang and Chen [8] are not listed in the present study. The measurement locations of Case B are farther away from the estimated surface than those listed in the work of Yang and Chen [8]. Note that the present problem is regarded as an inverse problem of parameter estimation in the work of Yang and Chen [8]. On the other hand, the functional form of the unknown surface temperature was given in advance and seven unknown parameters were estimated by the inverse analysis. But, in the present study, the functional form of the surface temperature is unknown a priori. Thus the functional form, as shown in Eq. (22), is applied to estimate the unknown surface temperature in each analysis interval.

Figs. 2–4, respectively, show the comparison of the surface temperature $T(1,y,t)$ between the exact results and the present results estimated from three and four measuring points of Case B in Table 1 for $t = 0.05, 0.15$ and 0.25 with respect to $\omega = 0$. The results show that the present numerical scheme has good accuracy even though thermocouples are located far away from the estimated surface. Their standard deviations of the mean with respect to the exact data σ^* are all around 0.1% for three measuring points. To show the accuracy and reliability of the estimates, more temperature measurements can be needed than the number of unknowns. Based on this concept, the unknown surface temperature is first

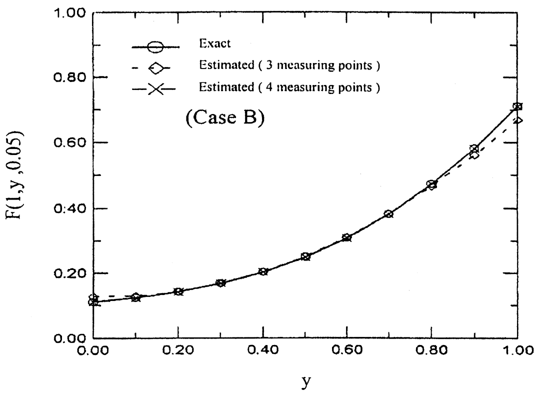

Fig. 2. Comparison of the present estimated $F(1, y, 0.05)$ with the exact one for Case B and $\omega = 0$.

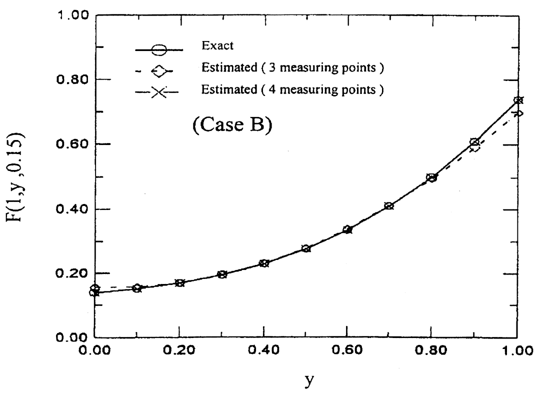

Fig. 3. Comparison of the present estimated $F(1, y, 0.15)$ with the exact one for Case B and $\omega = 0$.

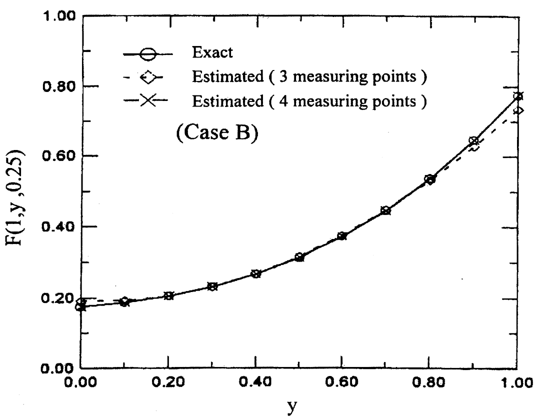

Fig. 4. Comparison of the present estimated $F(1, y, 0.05)$ with the exact one for Case B and $\omega = 0$.

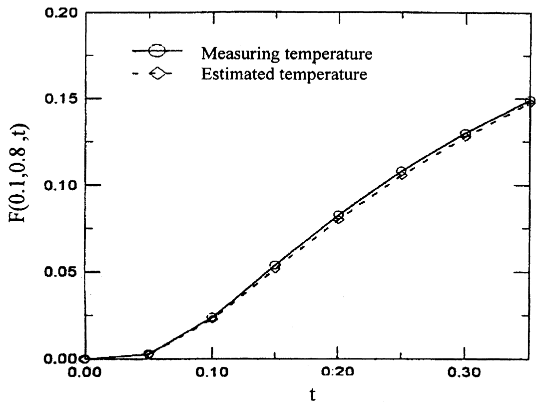

Fig. 5. Comparison of the estimated temperature $T^{est}(0.1, 0.8, t)$ and $T^{exa}(0.1, 0.8, t)$ for Case B and $\omega = 0$.

estimated by using three measuring points (0.1, 0.2), (0.1, 0.4) and (0.1, 0.6) and then the resulting estimates are used to determine the nodal temperature $T^{est}(0.1, 0.8, t)$. Thereafter, a comparison of the exact temperature $T^{exa}(0.1, 0.8, t)$ and present estimate $T^{est}(0.1, 0.8, t)$ is made, as shown in Fig. 5. It can be found that the agreement between them is very good. This comparative result further shows that the present hybrid method has good accuracy and good reliability.

To investigate the effect of the measurement error on the estimates, a comparison of the present estimates at $x = 1$ and results of Yang and Chen [8] for various ω values at various dimensionless times is made, as shown in Figs. 6–11. The surface temperature distributions at $x = 1$ shown in Figs. 6–11 are obtained by using the measuring points of Case B in Table 1. From these figures, the results of Yang and Chen [8] using three measuring points exhibit unstable behavior for larger ω values (i.e., $\omega = 3\%$ and 5%) and deviate from the exact results. These deviations can be improved by increasing the measuring points for the method of Yang and Chen [8]. However, it can be found from Fig. 11(a) that the estimates of Yang and Chen [8] still deviate from the exact results for $t = 0.25$. On the other hand, the present estimates exhibit stable behavior for various ω values and agree with the exact results even though three measuring points are used. In general, the estimation of the unknown surface temperature using four measuring points is slightly better than that using three measuring points. It is worth mentioning that the difference of the present estimates and the exact results does not increase with time. It can be observed from Figs. 2–4 and Figs. 6–11 that the effect of the measurement locations on the present estimates is not significant. This further implies that the present hybrid method can provide the good approximation for the estimated results even though the measurement locations are located far away from the estimated surface.

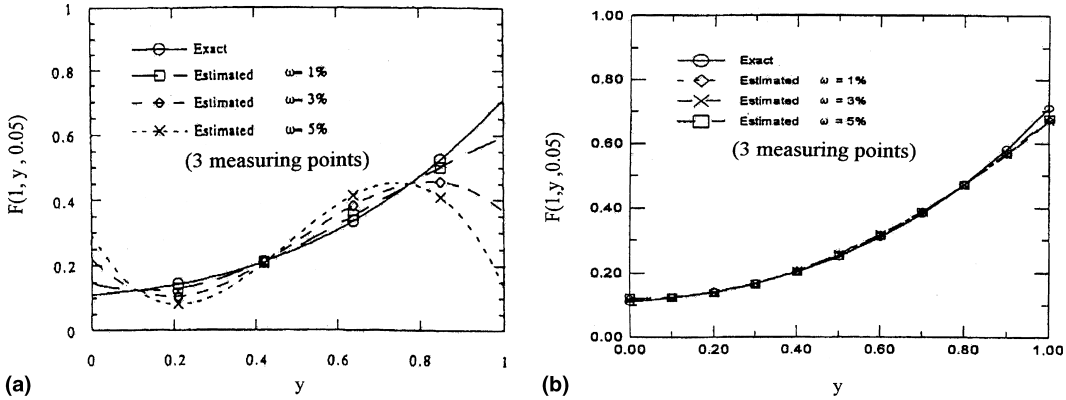

Fig. 6. Comparison of $F(1, y, 0.05)$ using three measuring points for various ω values: (a) Yang and Chen [8]; (b) present estimates with Case A.

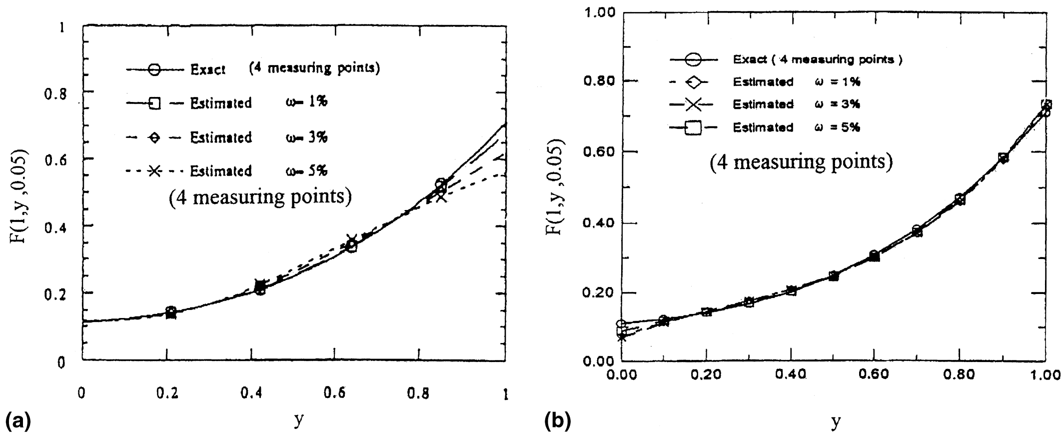

Fig. 7. Comparison of $F(1, y, 0.05)$ using four measuring points for various ω values: (a) Yang and Chen [8]; (b) present estimates with Case A.

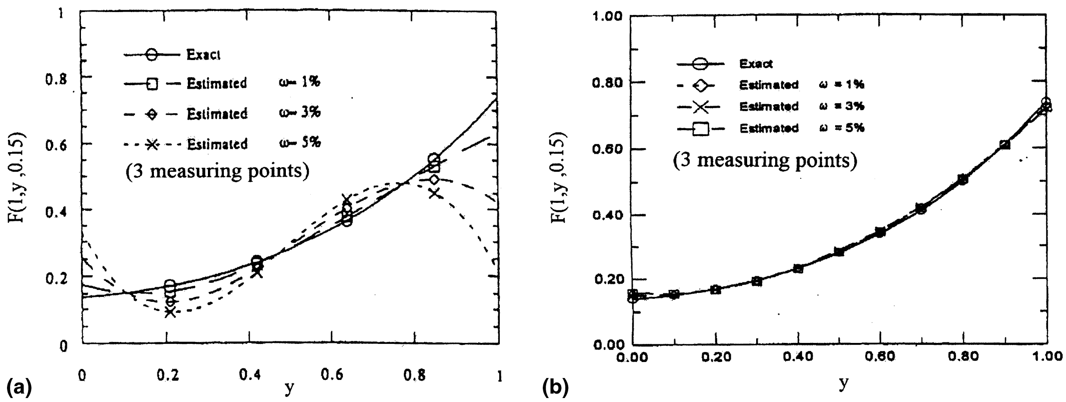

Fig. 8. Comparison of $F(1, y, 0.15)$ using three measuring points for various ω values: (a) Yang and Chen [8]; (b) present estimates with Case A.

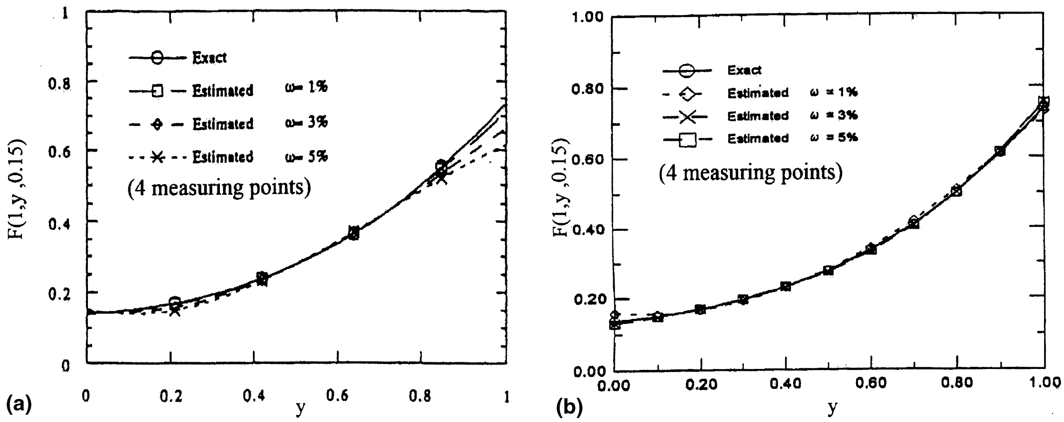

Fig. 9. Comparison of $F(1, y, 0.15)$ using four measuring points for various ω values: (a) Yang and Chen [8]; (b) present estimates with Case A.

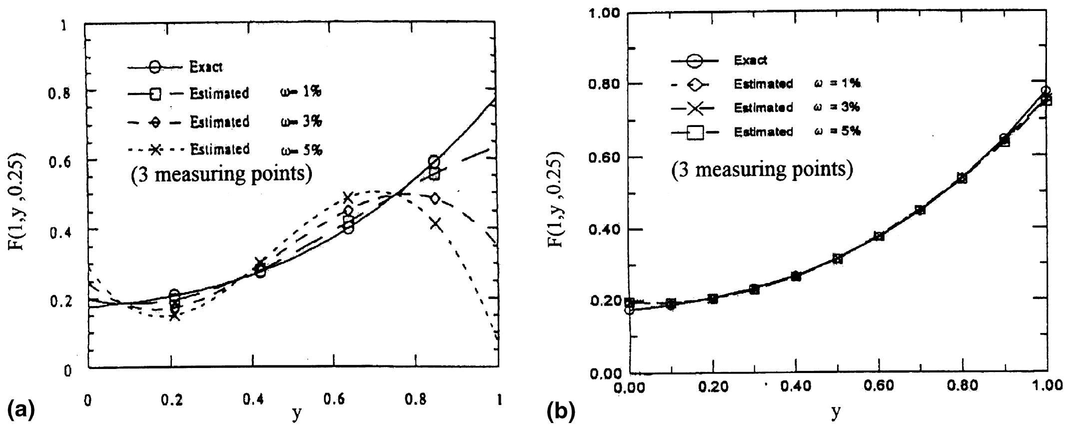

Fig. 10. Comparison of $F(1, y, 0.25)$ using three measuring points for various ω values: (a) Yang and Chen [8]; (b) present estimates with Case A.

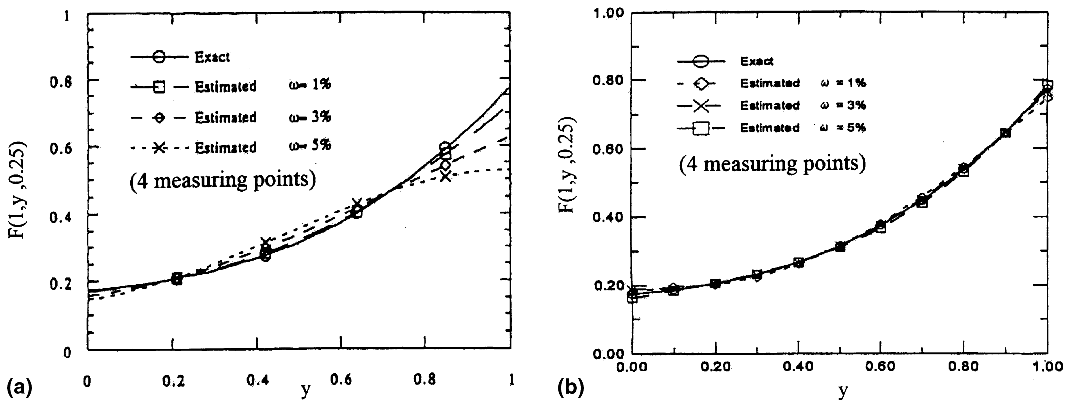

Fig. 11. Comparison of $F(1, y, 0.25)$ using four measuring points for various ω values: (a) Yang and Chen [8]; (b) present estimates with Case A.

4. Conclusion

The hybrid application of the Laplace transform and the FDM in conjunction with the least-squares scheme and a sequential-in-time concept is successfully applied to estimate the unknown surface temperature from temperature data measured at some locations in a plate. The functional form of the unknown surface temperature is unknown a priori. Since the present method is not a time-stepping procedure, the unknown surface temperature at any specific analysis sub-time interval can be predicted from the temperature measurements inside the plate without any step-by-step computations from $t = t_0$. The present estimates exhibit stable behavior for various ω values and agree with the exact results even though three measuring points are used. A small effect of the measurement locations on the estimates can be observed in the present study. This implies that the present hybrid method offers a great deal of flexibility.

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